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Strong $\Lambda\pi$ Phase Shifts for CP Violation in Weak $\Xi \rightarrow \Lambda\pi$ Decay[★]

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Abstract

Strong interaction $\Lambda\pi$ phase shifts relevant for the weak nonleptonic decay $\Xi \rightarrow \Lambda\pi$ are calculated using baryon chiral perturbation theory. We find in leading order that the S-wave phase shift vanishes and the $J = \frac{1}{2}$ P-wave phase shift is -1.7° . The small phase shifts imply that CP violation in this decay will be difficult to observe. Our results follow from chiral $SU(2)_L \times SU(2)_R$ symmetry.

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In the standard six-quark model CP violation arises from the phase δ in the Cabibbo-Kobayashi-Maskawa matrix. So far CP violation has only been observed in second order weak $K^0 - \bar{K}^0$ mixing and it is not known if this arises from the phase δ or from some new interaction associated with a very large mass scale. The latter possibility leads to a superweak scenerio for CP violation [1]. Observation of CP violation in a first order weak decay amplitude (sometimes referred to as direct CP violation) would rule out the superweak model as its sole origin. Avenues for detecting CP violation in first order weak decay amplitudes are the measurement of a nonzero value for the parameter ϵ' and the measurement of asymmetries in B decay. Recently it has been proposed to measure direct CP violation in the nonleptonic hyperon decay chain $\Xi \rightarrow \Lambda\pi \rightarrow p\pi\pi$ at Fermilab [2]. To observe CP violation in this case requires not only a CP violating phase in the weak hyperon decay amplitudes but also a phase from final state interactions. In this letter we calculate the strong $\Lambda\pi$ phase shifts that are important for observing CP violation in Ξ nonleptonic weak decay using baryon chiral perturbation theory. An interesting aspect of our work is that we are able to make predictions using only chiral $SU(2)_L \times SU(2)_R$ symmetry by utilizing the measured $\Sigma^- \rightarrow \Lambda e \bar{\nu}_e$ decay rate to determine the magnitude of the $\Sigma\Lambda\pi$ coupling. Strong phase shifts relevant for weak $\Xi \rightarrow \Lambda\pi$ decay were first calculated about 30 years ago and we compare our results with these earlier calculations [3-5]. One important difference between our approach and the previous work is that we don't rely on $SU(3)$ symmetry. We find that the S-wave phase shift vanishes at leading order in chiral perturbation theory and that the P-wave $J = \frac{1}{2}$ phase shift is only -1.7° . This suggests that CP violation in the recently proposed hyperon decay experiment at Fermilab will be dominated by the $\Lambda \rightarrow p\pi$ part of the decay chain.

Nonleptonic $\Xi \rightarrow \Lambda\pi$ decay is characterized by S-wave and P-wave amplitudes which we denote respectively by S and P . The differential decay rate (in the Ξ rest frame) has the form

$$\begin{aligned} \frac{d\Gamma}{d\Omega} \propto & 1 - \alpha (\hat{s}_\Xi \cdot \hat{p}_\pi + \hat{s}_\Lambda \cdot \hat{p}_\pi) + \beta \hat{p}_\pi \cdot (\hat{s}_\Xi \times \hat{s}_\Lambda) \\ & + \gamma \hat{s}_\Xi \cdot \hat{s}_\Lambda + (1 - \gamma) (\hat{s}_\Lambda \cdot \hat{p}_\pi) (\hat{s}_\Xi \cdot \hat{p}_\pi) \quad , \end{aligned} \quad (1)$$

where \hat{s}_Ξ and \hat{s}_Λ are unit vectors along the direction of the Ξ and Λ spins and \hat{p}_π is a unit vector along the direction of the pion momentum. The parameters α , β and γ which characterize the decay distribution are expressed in terms of the S-wave and P-wave amplitudes as follows;

$$\alpha = \frac{2 \operatorname{Re}(S^*P)}{|S|^2 + |P|^2} , \quad (2)$$

$$\beta = \frac{2 \operatorname{Im}(S^*P)}{|S|^2 + |P|^2} , \quad (3)$$

$$\gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2} . \quad (4)$$

The S-wave and P-wave amplitudes are complex numbers

$$S = |S|e^{i\delta_S} , \quad P = |P|e^{i\delta_P} . \quad (5)$$

In terms of the modulus and phase of S and P the parameters α and β are

$$\alpha = 2 \frac{|S||P|}{|S|^2 + |P|^2} \cos(\delta_S - \delta_P) , \quad (6)$$

$$\beta = -2 \frac{|S||P|}{|S|^2 + |P|^2} \sin(\delta_S - \delta_P) . \quad (7)$$

Isospin symmetry ensures that S and P for the decays $\Xi^- \rightarrow \Lambda\pi^-$ and $\Xi^0 \rightarrow \Lambda\pi^0$ are related by a factor of $\sqrt{2}$. The quantities δ_S and δ_P are respectively equal (up to a factor of π) to the strong interaction S-wave and $J = \frac{1}{2}$ P-wave $\Lambda\pi$ phase shifts plus small but important contributions from direct weak interaction CP violation [6].

The decay distribution for $\bar{\Xi} \rightarrow \bar{\Lambda}\pi$ is also given by eq.(1) with parameters $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ whose expression in terms of the S-wave and P-wave amplitudes \bar{S} and \bar{P} are similar to eqs.(2), (3) and (4). The only difference is that the analog of eqs.(2) and (3) have

a minus sign. The Λ 's produced in the decay of unpolarised Ξ 's have a polarisation α (as seen in eq.(1)) and an important measure of CP violation is the asymmetry

$$\mathcal{A} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} . \quad (8)$$

In terms of the phases of the S-wave and P-wave amplitudes the above becomes

$$\mathcal{A} = \frac{\cos(\delta_S - \delta_P) - \cos(\delta_{\bar{S}} - \delta_{\bar{P}})}{\cos(\delta_S - \delta_P) + \cos(\delta_{\bar{S}} - \delta_{\bar{P}})} . \quad (9)$$

We denote the $J = \frac{1}{2}$ S-wave and P-wave $\Lambda\pi$ phase shifts by δ_0 and δ_1 respectively and write

$$\delta_S - \delta_P = \delta_0 - \delta_1 + \phi_{CP} + \pi , \quad (10a)$$

$$\delta_{\bar{S}} - \delta_{\bar{P}} = \delta_0 - \delta_1 - \phi_{CP} + \pi , \quad (10b)$$

with ϕ_{CP} the phase that results from direct weak interaction CP violation. Data from $\Xi^- \rightarrow \Lambda\pi^-$ decay and $\Xi^0 \rightarrow \Lambda\pi^0$ decay give (neglecting CP violation) $\delta_0 - \delta_1 = (8 \pm 8)^\circ$ and $\delta_0 - \delta_1 = (38_{-19}^{+12})^\circ$ respectively. Putting eqs.(10) into eq.(9) yields

$$\mathcal{A} = -\tan \phi_{CP} \cdot \tan(\delta_0 - \delta_1) . \quad (11)$$

Eq.(11) indicates that the CP violating observable \mathcal{A} is small if the difference of phase shifts $\delta_0 - \delta_1$ is small. In this letter we calculate the strong interaction $\Lambda\pi$ phase shifts δ_0 and δ_1 using chiral perturbation theory.

In chiral perturbation theory the pions are incorporated into the 2×2 special unitary matrix

$$\Sigma = \exp(2iM/f) , \quad (12)$$

where

$$M = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix} , \quad (13)$$

and $f \simeq 132$ MeV is the pion decay constant. Under chiral $SU(2)_L \times SU(2)_R$ sym-

metry

$$\Sigma \rightarrow L \Sigma R^\dagger , \quad (14)$$

where $L \in SU(2)_L$ and $R \in SU(2)_R$. It is also convenient to introduce the square root of Σ ,

$$\xi = \exp(iM/f) , \quad (15)$$

which transforms under $SU(2)_L \times SU(2)_R$ as

$$\xi \rightarrow L \xi U^\dagger = U \xi R^\dagger . \quad (16)$$

Here U is a complicated nonlinear function of L , R and the pion fields M . The combinations of meson fields

$$(A^\mu)_a^b = \frac{i}{2}(\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi)_a^b , \quad (17a)$$

$$(V^\mu)_a^b = \frac{i}{2}(\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi)_a^b , \quad (17b)$$

play an important role in the interactions of pions with other fields. Note that V^μ contains terms with an even number of pion fields and A^μ contains terms with an odd number of pion fields.

The baryon fields we use are the spin $\frac{1}{2}$ isosinglet Λ , the spin $\frac{1}{2}$ isotriplet Σ_{ab} ($\Sigma_{11} = \Sigma^+$, $\Sigma_{12} = \Sigma_{21} = \frac{1}{\sqrt{2}}\Sigma^0$ and $\Sigma_{22} = \Sigma^-$) and the spin $\frac{3}{2}$ isotriplet $\Sigma_{ab}^{*\mu}$ (the assignment of the Σ^* 's to $\Sigma_{ab}^{*\mu}$ is analogous to the assignment of the Σ 's to Σ_{ab}). Under chiral $SU(2)_L \times SU(2)_R$ these fields transform as

$$\Lambda \rightarrow \Lambda , \quad (18a)$$

$$\Sigma_{ab}^{(*\mu)} \rightarrow U_{ac} U_{bd} \Sigma_{cd}^{(*\mu)} , \quad (18b)$$

where repeated roman indices a, b, \dots are summed over 1, 2. Strong interactions of these baryons with pions are described by a chiral Lagrangian that is invariant under

parity and chiral $SU(2)_L \times SU(2)_R$ symmetry. Expanding in derivatives this chiral Lagrangian density is [7,8],

$$\mathcal{L} = \mathcal{L}_\Lambda + \mathcal{L}_\Sigma + \mathcal{L}_{\Sigma^*} + \mathcal{L}_{int} , \quad (19)$$

where

$$\mathcal{L}_\Lambda = \bar{\Lambda} i v \cdot \partial \Lambda, \quad (20a)$$

$$\mathcal{L}_\Sigma = \bar{\Sigma}^{ab} i v \cdot \partial \Sigma_{ab} + 2 \bar{\Sigma}^{ab} v \cdot V_a^c \Sigma_{cb} + (m_\Lambda - m_\Sigma) \bar{\Sigma}^{ab} \Sigma_{ab} , \quad (20b)$$

and

$$\mathcal{L}_{int} = g_{\Sigma\Lambda} \bar{\Lambda} S \cdot A_a^b \Sigma_{cb} \epsilon^{ac} + g_{\Sigma^*\Lambda} \bar{\Lambda} A_a^b \cdot \Sigma_{bc}^* \epsilon^{ac} + h.c. . \quad (20c)$$

The expression for \mathcal{L}_{Σ^*} is similar to eq.(20b). In eqs.(20) S is the spin operator four-vector, ϵ^{ac} is the antisymmetric tensor, $\epsilon^{11} = \epsilon^{22} = 0$, $\epsilon^{12} = -\epsilon^{21} = 1$, and v is the baryon four-velocity. There are also interaction terms with one derivative involving two $\Sigma^{(*)}$ fields and an odd number of pions. However these interactions are not needed for our computation. We treat $m_\Sigma - m_\Lambda$ and $m_{\Sigma^*} - m_\Lambda$ as small quantities. For power counting purposes these mass differences are considered to be the same order as a single derivative.

The magnitude of the couplings $g_{\Sigma^*\Lambda}$ and $g_{\Sigma\Lambda}$ can be determined from experiment. Comparison of the measured $\Sigma^{*+} \rightarrow \Lambda \pi^+$ decay width with

$$\Gamma(\Sigma^{*+} \rightarrow \Lambda \pi^+) = g_{\Sigma^*\Lambda}^2 \frac{1}{6\pi} \frac{|\vec{p}_\pi|^3}{f^2} \frac{m_\Lambda}{m_{\Sigma^*}} , \quad (21)$$

gives $g_{\Sigma^*\Lambda}^2 \simeq 1.49$. There is a Goldberger-Treiman type relation that relates matrix elements of the axial current to the $\Sigma\Lambda\pi$ coupling $g_{\Sigma\Lambda}$. Using the Noether procedure we find that in chiral perturbation theory matrix elements of the left-handed current

are given by

$$\bar{u}\gamma^\mu(1 - \gamma_5)d = g_{\Sigma\Lambda} \bar{\Lambda} S^\mu \Sigma^- + \dots \quad (22)$$

In eq.(22) the ellipses denote pieces involving other baryon fields, the pion fields and terms with derivatives. The resulting $\Sigma^- \rightarrow \Lambda e \bar{\nu}_e$ decay rate is

$$\Gamma(\Sigma^- \rightarrow \Lambda e \bar{\nu}_e) = \frac{G_F^2}{80\pi^3} |V_{ud}|^2 g_{\Sigma\Lambda}^2 (m_\Sigma - m_\Lambda)^5. \quad (23)$$

Comparing with the measured $\Sigma^- \rightarrow \Lambda e \bar{\nu}_e$ decay rate yields $g_{\Sigma\Lambda}^2 \simeq 1.44$.

The Feynman diagrams in Fig.1 determine the S-wave and $J = \frac{1}{2}$ P-wave $\Lambda\pi$ phase shifts at the leading order of chiral perturbation theory. As a function of the pion energy in the center of mass frame,^{*} E_π , we find the S-wave phase shift to be

$$\delta_0(E_\pi) = 0, \quad (24)$$

and the $J = \frac{1}{2}$ P-wave phase shift to be

$$\delta_1(E_\pi) = -\frac{(E_\pi^2 - m_\pi^2)^{3/2}}{12\pi f^2} \cdot \left[\frac{1}{4} \frac{g_{\Sigma\Lambda}^2}{E_\pi + m_\Sigma - m_\Lambda} + \frac{3}{4} \frac{g_{\Sigma\Lambda}^2}{E_\pi + m_\Lambda - m_\Sigma} - \frac{4}{3} \frac{g_{\Sigma^*\Lambda}^2}{E_\pi + m_{\Sigma^*} - m_\Lambda} \right]. \quad (25)$$

The expression inside the bracket for $\delta_1(E_\pi)$ is singular at the unphysical pion energy $E_\pi = m_\Sigma - m_\Lambda$ because of the Σ pole. (When the energy is near this value other terms we have neglected become important and tame the singularity.) Note that there is no singularity at $E_\pi = m_{\Sigma^*} - m_\Lambda$ as there is no Σ^* pole in the $J = \frac{1}{2}$ channel. The S-wave phase shift vanished because Λ is an isospin-zero baryon (hence there is no coupling to two pions in \mathcal{L}_Λ) and because the $\Sigma\Lambda\pi$ and $\Sigma^*\Lambda\pi$ interactions

^{*} In heavy baryon chiral perturbation theory the centre of mass frame and the baryon rest frame coincide.

have the pions derivatively coupled. At higher order in chiral perturbation theory we expect an S-wave phase shift suppressed by a factor of order E_π/Λ_χ (where Λ_χ is the chiral symmetry breaking scale) compared, for example, with the S-wave pion-nucleon phase shifts. At $E_\pi \sim 200\text{MeV}$ the pion-nucleon S-wave phase shifts are several degrees. A contribution to the S-wave $\Lambda\pi$ phase shift suppressed by E_π/Λ_χ arises from higher derivative terms, eg.

$$\mathcal{L}_{\text{higher}} = \frac{c}{\Lambda_\chi} \bar{\Lambda}\Lambda A_b^a \cdot A_a^b \quad . \quad (26)$$

The coefficient c is expected to be of order unity, but as it is an unknown quantity the S-wave phase shift at this order is not calculable. Previous calculations [5] did not find a small S-wave $\Lambda\pi$ phase shift. Fig.2 contains a plot of the $J = \frac{1}{2}$ P-wave phase shift δ_1 as a function of E_π . For the hyperon decay $\Xi \rightarrow \Lambda\pi$ we need the phase shift evaluated at $E_\pi \simeq m_\Xi - m_\Lambda = 206\text{MeV}$. At this energy the $J = \frac{1}{2}$ P-wave phase shift is $\delta_1 = -1.7^\circ$. This is within a factor of two of the value for δ_1 obtained in previous calculations [3,5].

Our predictions for the phase shifts do not make use of chiral $SU(3)_L \times SU(3)_R$ symmetry. However, chiral perturbation theory is an expansion in E_π and our result for $\delta_1(m_\Xi - m_\Lambda)$ relies on $m_\Xi - m_\Lambda$ and hence the strange quark mass being small compared with the chiral symmetry breaking scale.

The smallness of the $J = \frac{1}{2}$ P-wave phase shift $\delta_1(E_\pi)$ is partly the result of a cancellation between the Feynman diagrams involving the Σ and Σ^* . This cancellation becomes exact in the large N_c limit [8,9] where $m_\Sigma = m_{\Sigma^*} = m_\Lambda$ and $g_{\Sigma^*\Lambda}^2 = \frac{3}{4}g_{\Sigma\Lambda}^2$.

Our calculations indicate that for the weak decay $\Xi \rightarrow \Lambda\pi$ the difference between the S- and $J = \frac{1}{2}$ P-wave phase shifts $\delta_0 - \delta_1$ and consequently the CP violating asymmetry \mathcal{A} are small. Therefore, it is likely that any CP violation observed in the recently proposed Fermilab experiment will be dominated by CP violation in the $\Lambda \rightarrow p\pi$ part of the $\Xi \rightarrow \Lambda\pi \rightarrow p\pi\pi$ decay chain.

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Figure Captions

Figure 1. Feynman diagrams contributing to the $J = \frac{1}{2}$ P-wave $\Lambda\pi$ phase shift δ_1 . There is no contribution to the S-wave phase shift at leading order in chiral perturbation theory.

Figure 2. The $J = \frac{1}{2}$ P-wave phase shift δ_1 (in degrees) as a function of pion energy E_π (in MeV).

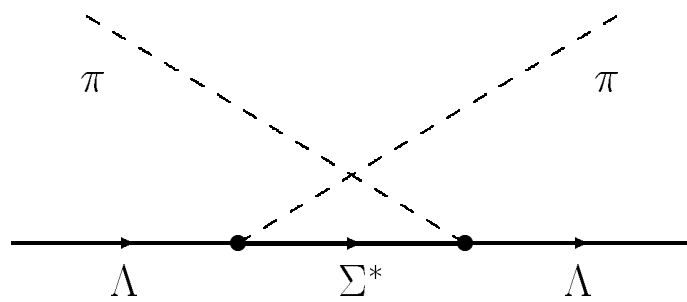
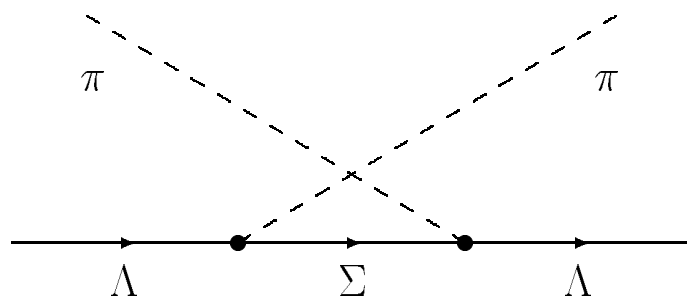
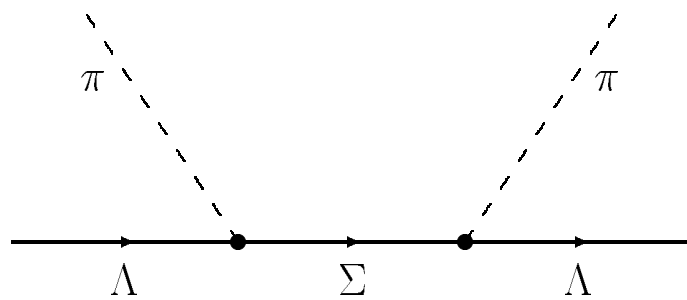


Figure 1

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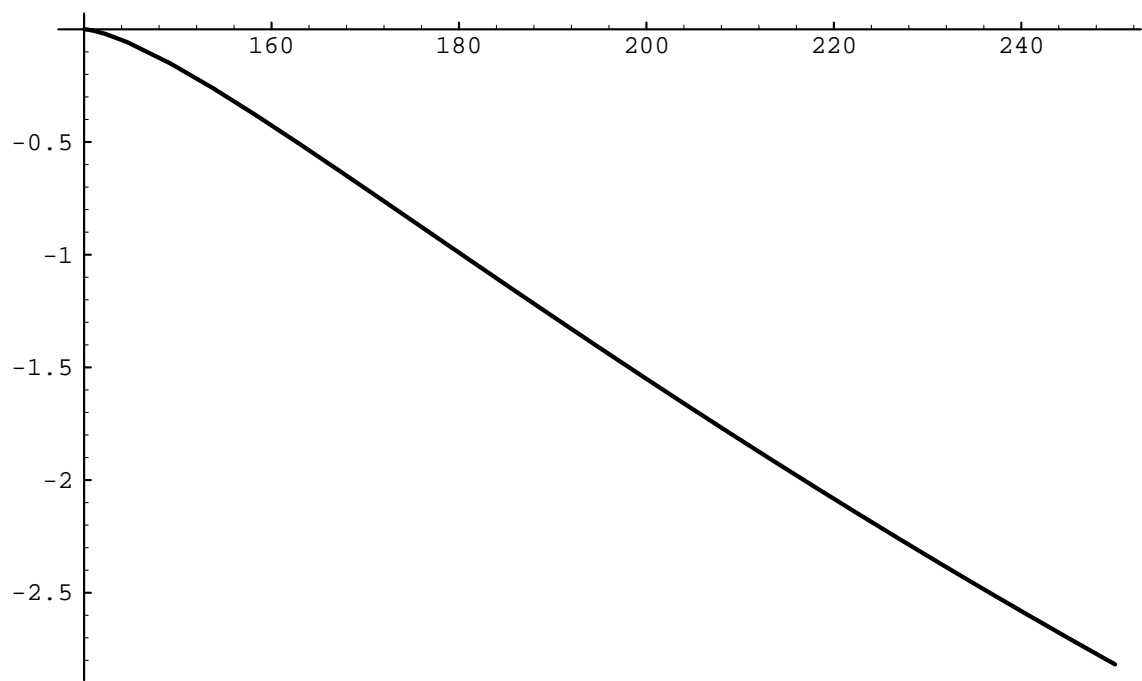


Figure 2

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